< 정답 및 해설 >

<정답>

문항	1	2	3	4	5	6	7	8	9	10	11	12	13
정답	③	②	①	④	②	②	④	③	④	①	①	④	③
문항	14	15	16	17	18	19	20	21	22	23	24	25	
정답	②	④	①	②	②	④	②	①	④	④	③	③	

<해설>

1. $\tan^{-1}(-2) = A\,,\ -\frac{\pi}{2} < A < 0$

$\therefore\ \sec(\tan^{-1}(-2)) = \sec A = \sqrt{5}$

2. $\vec{a}\cdot\vec{b}\times\vec{c} = \begin{vmatrix} 2 & -1 & 0 \\ 0 & 3 & 1 \\ 5 & 0 & 2 \end{vmatrix}$

$= 2(6-0) + (0-5) = 7$

3. $0 \le \left|\ \frac{(x+1)\sin(x^2)}{x^2+1}\ \right| \le \left|\ \frac{x+1}{x^2+1}\ \right|\ ,\ \lim_{x\to\infty}\frac{x+1}{x^2+1} = 0$

$\therefore\ \lim_{x\to\infty}\frac{(x+1)\sin(x^2)}{x^2+1} = 0$

4. $\lim_{x\to 0+}\frac{\int_{x^2}^{x^4}\sinh\sqrt{t}\,dt}{x^3} = \lim_{x\to 0+}\frac{4x^3\sinh x^2 - 2x\sinh x}{3x^2}$

$= \lim_{x\to 0+}\frac{8x\sinh x^2 + 8x^3\cosh x^2 - 2\cosh x}{3}$

$= -\frac{2}{3}$

5. $x=0$일 때, $y=-\frac{1}{3}$.

$2xy+x^2y'+\sec^2(\ln(xy+1))\frac{y+xy'}{xy+1}=-(y^2+2xyy')\sin(xy^2)+3y'$

$x=0,\ y=-\frac{1}{3}$일 때, $y'=-\frac{1}{9}$

$\therefore$ 접선의 방정식은 $y=-\frac{1}{3}-\frac{1}{9}x$.

6. $\sqrt{x-1}=t \Rightarrow x-1=t^2$

$$\int_1^2 \frac{\sqrt{x-1}+1}{x}\,dx=\int_0^1 \frac{t+1}{t^2+1}\,2t\,dt=\int_0^1 \frac{2(t^2+1)+2t-2}{t^2+1}\,dt$$
$$=\int_0^1\left(2+\frac{2t}{t^2+1}-\frac{2}{t^2+1}\right)dt=2+\ln 2-\frac{\pi}{2}$$

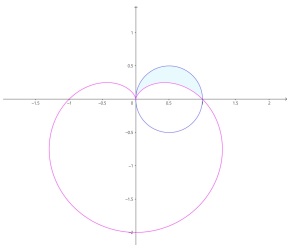
7. $\int_0^{\pi/2}\left(3(1-\cos^2 x)\sin x-\frac{1+\cos 4x}{2}\right)dx$

$=\left[-3\cos x+\cos^3 x-\frac{1}{2}x-\frac{\sin 4x}{8}\right]_0^{\pi/2}=2-\frac{\pi}{4}$

8. $\frac{dy}{dx}=\frac{3t^2-2}{2t}$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d}{dt}\left(\frac{3t^2-2}{2t}\right)\frac{dt}{dx}=\frac{6t^2+4}{4t^2}\frac{1}{2t}\Bigg|_{t=1}=\frac{5}{4}$$

9. (넓이) = $\frac{1}{2}\int_0^{\pi/2}(\cos^2\theta-(1-\sin\theta)^2)\,d\theta=1-\frac{\pi}{4}$



10. $\sin x=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}x^{2n+1}$

$\therefore\ 3\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}\left(\frac{\pi}{3}\right)^{2n+1}=3\sin\left(\frac{\pi}{3}\right)=\frac{3\sqrt{3}}{2}$

11. $F(x,y,z)=e^{x^2yz}+xy^2z+\ln(z+xy)-1=0$,
$x=0, y=1$이면 $z=1$

$$\left.\frac{\partial z}{\partial x}\right|_{(0,1)} = -\frac{F_x(0,1,1)}{F_z(0,1,1)} = -\left.\frac{2xyz\,e^{x^2yz}+y^2z+\dfrac{y}{z+xy}}{x^2ye^{x^2yz}+xy^2+\dfrac{1}{z+xy}}\right|_{(0,1,1)} = -\frac{1+1}{1} = -2$$

12. $\vec{u} = \dfrac{\vec{v}}{\|\vec{v}\|} = \dfrac{1}{5}\langle 3,4\rangle$

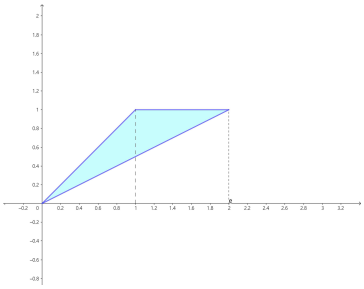
$$D_{\vec{u}}f(1,1) = \nabla f(1,1)\cdot\frac{1}{5}\langle 3,4\rangle = \left\langle \frac{y}{1+x^2y^2}, \frac{x}{1+x^2y^2}\right\rangle_{(1,1)}\cdot\frac{1}{5}\langle 3,4\rangle$$

$$= \frac{1}{2}\langle 1,1\rangle\cdot\frac{1}{5}\langle 3,4\rangle = \frac{7}{10}$$

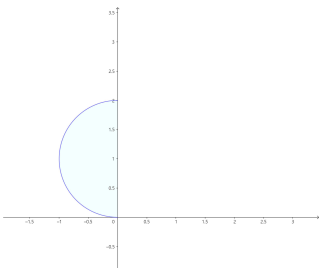
13. $f_r = f_x x_r + f_y y_r$
$f_{rr} = f_{xx}(x_r)^2 + f_{yy}(y_r)^2 + 2f_{xy}x_r y_r + f_x x_{rr} + f_y y_{rr}$
$x_r = 2,\ x_{rr} = 2,\ y_r = 2,\ y_{rr} = 0$

$\therefore\ f_{rr} = 4f_{xx} + 4f_{yy} + 8f_{xy} + 2f_x = 12$

14. $$\int_0^1\int_y^{2y} e^{-y^2}\,dx\,dy = \int_0^1 y e^{-y^2}\,dy = \frac{1}{2}(1-e^{-1})$$

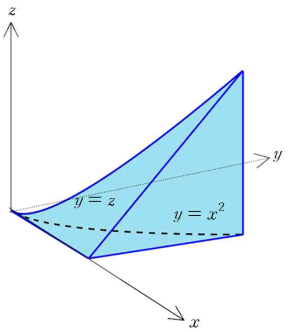


15. $$\int_{\pi/2}^{\pi}\int_0^{2\sin\theta} r^2\,dr\,d\theta = \int_0^{\pi/2}\frac{8}{3}\sin^3\theta\,d\theta = \frac{16}{9}$$



16. $\int_0^1 \int_0^y \int_{\sqrt{y}}^1 f(x,y,z)\,dx\,dz\,dy = \int_0^1 \int_0^{x^2} \int_a^b f(x,y,z)\,dy\,dz\,dx$

$z \le y \le x^2 \;\therefore\; a = z,\; b = x^2$



17. $\frac{x}{1+x^2}dx = \frac{1}{\sqrt{1-y^2}}dy \;\Rightarrow\; \int \frac{x}{1+x^2}dx = \int \frac{1}{\sqrt{1-y^2}}dy + C$

$\frac{1}{2}\ln(1+x^2) = \sin^{-1}y + C,\; y(0) = 1, \quad C = -\frac{\pi}{2}$

$\therefore \quad \sin^{-1}y - \frac{1}{2}\ln(1+x^2) = \frac{\pi}{2}$

18. 표준형 $\frac{dy}{dx} + \cos x\, y = \sin x \cos x$

적분인자: $e^{\int \cos x\,dx} = e^{\sin x}$,

$(e^{\sin x}y)' = e^{\sin x}\sin x\cos x$

$e^{\sin x}y = \int e^{\sin x}(\cos x \sin x)\,dx + C = e^{\sin x}(\sin x - 1) + C$

$\therefore\; y = \sin x - 1 + Ce^{-\sin x}$

19. $P = y^2 + xy^3,\; Q = 5y^2 - xy + y^3\sin y$

적분인자 : $F(y) = \exp\int \frac{1}{P}(Q_x - P_y)\,dy = \exp\int \frac{-y-(2y+3xy^2)}{y^2+xy^3}dy = y^{-3}$

$\left(\frac{1}{y} + x\right)dx + \left(\frac{5}{y} - \frac{x}{y^2} + \sin y\right)dy = 0$

$\therefore \quad \frac{x}{y} + \frac{x^2}{2} + 5\ln y - \cos y = C$

20. $y'' + \frac{2x}{1+x^2}y' = 0 \;,\; p(x) = \frac{2x}{1+x^2}$

$y_2 = u(x)y_1 \Rightarrow\; u(x) = \int \frac{e^{-\int p\,dx}}{y_1^2}dx = \int \frac{e^{-\int \frac{2x}{1+x^2}dx}}{y_1^2}dx = \int \frac{e^{-\ln(1+x^2)}}{1}dx$

$$= \int \frac{1}{1+x^2}\,dx \quad = \tan^{-1}x$$

$\therefore\ y_2 = \tan^{-1}x$

21. (제차해) $m^2+4=0 \Rightarrow m = \pm 2i$, $y_h = C_1\cos 2x + C_2 \sin 2x$

(특별해) $y_p = u_1(x)\cos 2x + u_2(x)\sin 2x$

$$W[\cos 2x, \sin 2x] = \begin{vmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix} = 2$$

$$u_1' = \frac{\begin{vmatrix} 0 & \sin 2x \\ 4\sec 2x & 2\cos 2x \end{vmatrix}}{2} = -2\tan 2x, \quad u_2' = \frac{\begin{vmatrix} \cos 2x & 0 \\ -2\sin 2x & 4\sec 2x \end{vmatrix}}{2} = 2$$

$$u_1(x) = -\int 2\tan 2x\,dx = \ln|\cos 2x|, \ u_2(x) = \int 2\,dx = 2x$$

$$y_p = \cos 2x \ln|\cos 2x| + 2x\sin 2x$$

$\therefore\ y = C_1\cos 2x + C_2\sin 2x + (\ln|\cos 2x|)\cos 2x + 2x\sin 2x$

22. (제차해) $m(m-1)-2m+2=(m-1)(m-2)=0 \quad \therefore y_h = C_1 x + C_2 x^2$

(특별해) $y_p = u_1(x)x + u_2(x)x^2$

표준형 $y'' - \frac{2}{x}y' + \frac{2}{x^2}y = 3.$

$$W[x, x^2] = \begin{vmatrix} x & x^2 \\ 1 & 2x \end{vmatrix} = x^2$$

$$u_1' = \frac{\begin{vmatrix} 0 & x^2 \\ 3 & 2x \end{vmatrix}}{x^2} = -3, \ u_2' = \frac{\begin{vmatrix} x & 0 \\ 1 & 3 \end{vmatrix}}{x^2} = \frac{3}{x}$$

$$u_1 = \int -3\,dx = -3x, \ u_2 = 3\int \frac{1}{x}\,dx = 3\ln x$$

$\therefore\ y_p = (-3x)x + (3\ln x)x^2 = 3x^2(\ln x - 1)$

(일반해) $y = C_1 x + C_2 x^2 + 3x^2(\ln x - 1)$

$-1 = y(1) = C_1 + C_2 - 3 \quad \Rightarrow \quad C_1 + C_2 = 2$

$-1 = y'(1) = C_1 + 2C_2 - 3 \Rightarrow C_1 - 2C_2 = 2$

$\therefore\ C_1 = 2\ ,\ C_2 = 0$

$\therefore\ y = 2x + 3x^2(\ln x - 1)$

23. $f(t) = \cos\left(\frac{t}{2}\right)u(t-\pi) = -\sin\frac{1}{2}(t-\pi)\,u(t-\pi)$

$$\therefore\ \mathcal{L}\{f(t)\} = \mathcal{L}\left\{-\sin\frac{1}{2}(t-\pi)\,u(t-\pi)\right\} = -\frac{\frac{1}{2}}{s^2+\frac{1}{4}}e^{-\pi s} = -\frac{2}{4s^2+1}e^{-\pi s}$$

24. $\mathcal{L}^{-1}\left\{\dfrac{4}{(s^2+4)^2}\right\} = \sin 2t * \sin 2t = \dfrac{\sin 2t}{4} - \dfrac{t\cos 2t}{2}$

$\therefore\ \mathcal{L}^{-1}\left\{\dfrac{1}{(s^2-2s+5)^2}\right\} = e^t\left(\dfrac{\sin 2t}{4} - \dfrac{t\cos 2t}{2}\right)$

25. $f(t) = -\cos(t-\pi)\,u(t-\pi)$

$s^2 Y(s) - s\,y(0) - y'(0) + 9\,Y(s) = -\dfrac{s}{s^2+1}e^{-\pi s}$

$(s^2+9)\,Y(s) = s + 1 - \dfrac{s}{s^2+1}e^{-\pi s}$

$Y(s) = \dfrac{s+1}{s^2+9} - \dfrac{s}{(s^2+9)(s^2+1)}e^{-\pi s}$

$\mathcal{L}^{-1}\left(\dfrac{s+1}{s^2+9}\right) = \cos 3t + \dfrac{1}{3}\sin 3t$

$\mathcal{L}^{-1}\left\{\dfrac{-s}{(s^2+9)(s^2+1)}\right\} = \dfrac{1}{8}(\cos 3t - \cos t)$

$\mathcal{L}^{-1}\left\{\dfrac{-s}{(s^2+9)(s^2+1)}e^{-\pi s}\right\} = \dfrac{1}{8}(\cos 3(t-\pi) - \cos(t-\pi))\ u(t-\pi)$

$\therefore\ y(t) = \cos 3t + \dfrac{1}{3}\sin 3t + \dfrac{1}{8}(\cos 3(t-\pi) - \cos(t-\pi))\ u(t-\pi)$

$= \cos 3t + \dfrac{1}{3}\sin 3t + \dfrac{1}{8}(\cos t - \cos 3t)\,u(t-\pi)$